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Metaformal Realism and the Ontological Problematic

(Interregnum)

(Chapter 5 of Draft MS: *The Logic of Being: Heidegger, Truth, and Time*)

"Not empiricism and yet realism in philosophy, that is the hardest thing." -Wittgenstein

"A human is that being which prefers to represent itself within finitude, whose sign is death, rather than knowing itself to be entirely traversed and encircled by the omnipresence of infinity." -Badiou

I

In his 1951 Gibbs lecture, "Some basic theorems on the foundations of mathematics and their philosophical implications," drawing out some of the "philosophical consequences" of his two incompleteness theorems and related results, Kurt Gödel outlines a disjunctive alternative which, as I shall argue here, captures in a precise way the situation of contemporary ontology in its ongoing consideration of the relationship of formalism to the real of being:

> Either mathematics is incompletable in [the] sense that its evident axioms can never be comprised in a finite rule, i.e. to say the human mind (even within the realm of pure mathematics) infinitely surpasses the powers of any finite machine, or else there exist absolutely unsolvable Diophantine problems of the type specified…(where the case that both terms of the disjunction are true is not excluded so that there are, strictly speaking, three alternatives).[1]

A consequence of this aporeatic situation of contemporary thought, as I shall try to show, is that the longstanding philosophical debate over the relative priority of thought and being that finds expression in discusssions of "realism" and "anti-realism" (whether of idealist, positivist, or conventionalist forms) can only be assayed from the position of what I shall call a *meta-formal* reflection on the relationship of the *forms* of thought to the *real* of being. This isexactly the kind of reflection exemplified by Gödel's argument in the Gibbs lecture. Moreover, if Gödel's argument is correct, and if it bears (as I shall try to show it does) not only on the question of "mathematical reality" narrowly conceived but, more generally, on the very "relationship" of thought and being that is at issue in these discussions, it is also not neutral on this question of relative priority, but rather suggests a (necessarily disjunctive) kind of

[1] Gödel (1951), p. 134.

realism – what I shall call "metaformal" realism -- that differs markedly both from "metaphysical realism" and from the newer varieties of "speculative realism" on offer today.

The type of realism I shall defend here is not primarily a realism *about* any particular class or type of objects or entities. Thus it is not, *a fortiori*, an *empirical* realism or a naturalism (although I also do not think it is *inconsistent* with positions that march under these banners).[2] In particular, its primary source is not any empirical experience but rather the experience of formalization, both insofar as this experience points to the real-impossible point of the *actual relation* of thinkable forms to being and insofar as it schematizes, in results such as Gödel's, the intrinsic capacity of formalization problematically to capture and decompose its *own* limits. As such a position on the form of the possible relationship of thought to being, it is (as I shall argue here) relevant to the "ontological" problematic of the possible thought of being "itself", and is even requisite for a furtherance of this problematic on realist terms today. In particular, because meta-formal realism is not an ontic realism about any particular domain of entities, but rather unfolds the inherent structural forms of thought in relation to the sense of "being", it can support an *ontological* realism that is appropriate to the formal indication of the "relationship" of ontological difference between being and beings and also to the formally indicated problematic of the underlying structure of time as it is "given" in relation to "being as such."

Such an ontological realism is, as I shall argue here and in the following chapters, also requisite in order to develop the ontological problematic of sense and time in abeyance of any reduction of it to psychology, anthropology, or anthropologically grounded "cultures" or "social practices." Whether or not one agrees with the exegetical claim that Heidegger himself, after *Being and Time,* increasingly seeks to distance himself from and repudiate a residual "anthropologism" or humanism that still finds expression there in the "preparatory" fundamental analysis of Dasein (a repudiation that appears to find expression, for instance, in the forceful terms of the 194- "Letter on Humanism"), the sort of realism that I argue for here is at least a *possible* position relevant to the ontological problematic, both as developed in *Being and Time* and as it yields Heidegger's later interrogation of the "truth" of Being independently of any relation to entities. As such, as I shall argue, it is requisite (probably uniquely) for a realist ontological conception of time that avoids any derivation of time from the constitutive capacities of the representing and thinking subject. It also provides a concrete formal basis for critical arguments and positions that are unmistakably Heidegger's own. For on one hand, as I shall argue, the attitude or position of meta-formal realism as I shall develop it here provides a formal basis for the critique of any position that puts the *representing subject* at the basis of the possible thought of being by indicating the formal-ontological configuration that first underlies the ontological possibility of there being anything like a subject to begin with. On the other hand, and on the same realist terms, it provides a concrete basis for the critique of the identification of being with effective *actuality* [Wirklichkeit], that Heidegger sees as deeply characteristic of metaphysical thought and practice, most of all in its contemporary culmination in the regime of technology and totalizing "enframing" [Gestell].

In *The Politics of Logic*, I systematically interrogated the consequences of formalism and formalization in this sense for contemporary political, social, and intersubjective life according to the various

[2] I return to the issue of the relationship of realism to materialism in section IV, below.

orientations possible today for thought in its total relation to being, seeking to locate, in each case, the actual point and limits of the effective formal capture of the real in thought. In particular, I suggested there that both of the orientations I presented as "post-Cantorian" demand a *realist* attitude grounded in this experience of the transit of forms, and capable of acknowledging their inherent difference from anything simply *created* or *produced* by finite human thought. As the name "post-Cantorian" is meant to index, one decisive basis for this distinctive kind of realism is the chain of consequences following from the Cantorian event and the problematic accessibility of the infinite to mathematical thought, up to and including Gödel's incompleteness results, as these consequences offer to challenge and reconfigure the traditional conception of the human as an essentially finite agent of thought. As I also already suggested there, the metaformal realism I shall develop more fully here might be formulated precisely, referring in passing to the Lacanian motto according to which "the Real is the impasse of formalism," as a realism *of* the "Real" in something like Lacan's sense. The "Real," in this sense, is thought as one of the three "registers" of psychological development; here it is both an inherent limit-point and an obscurely constitutive underside for both of the other two "registers" of the Imaginary and the Symbolic and structurally articulates the subject's necessarily displaced or "barred" position in relation to what Lacan characterizes as the "thing" or the "object small a".[3] Thought in this way, the "real" in this sense is to be sharply distinguished both from "reality" in the sense of actuality and from any realm, regime or domain of actually existing objects. But as Lacan himself occasionally suggests, the problematic of "access" to the Real, at the structurally necessary point of the symbolic impasse which is, for him, formally constitutive of the very structure of the subject in the order of the symbolic, is by no means unrelated to the "ontological" problematic of the structural "place" of being as such in relation to the factical life and structured language of the being that thinks. As I shall argue here, this problematic, first developed (in *Being and Time*) as that of the constitutive structure of the kind of entity – Dasein – that is ontic-ontological in its constitutive relationship to being itself, and later (after the mid-1930s) as that of the truth of being in itself that first makes possible something like a "clearing" in which being can come to light as time, is one that both suggests and demands the rigorously formal realism that I shall defend, on partially independent grounds.

To arrive at the disjunctive conclusion he draws in the lecture, Gödel draws centrally on a concept central to twentieth-century inquiry into the foundations of mathematics, that of a "finite procedure." Such a procedure is one that can be carried out in finite number of steps by a system governed by well-defined and finitely stateable rules, a so-called "formal system." As Gödel points out, there are several rigorous ways to define such a system, but they have all been shown to be equivalent to the definition given by Turing of a certain specifiable type of machine (what has come to be called a "Turing machine").[4] The significance of the investigation of formal systems for research into the structure of

[3] Of course, Lacan's concept of the "Real" is complex and undergoes many changes of specification and inflection over the course of his career. I do not take a view here about how precisely to define it or which formulation is most important, but seek only to preserve the link that is constitutive for Lacan between the Real and formalization at the latter's point of inherent impasse. For a very exhaustive and illuminating treatment of Lacan's concept, see Eyers (2012). I also discuss Lacan's motto and Badiou's reversal of it into his own claim for a "theory of the pass of the real, in the breach opened up by formalization…" in Livingston (2012), pp. 188-192.

[4] This is a formulation of the "Church-Turing" thesis, which holds that the structure of a Turing machine (or any of several provably equivalent formulations) captures the 'intuitive' notion of solvability or effective computability.

mathematical cognition and reality lies in the possibility it presents of rigorously posing general questions about the capacities of such systems to solve mathematical problems or prove mathematical truths. For instance, one can pose as rigorous questions i) the question whether such a system is capable of proving *all* arithmetic truths about whole numbers; and ii) whether such a system is capable of proving a statement of its own consistency. Notoriously, Gödel's first and second incompleteness theorems, respectively, answer these two questions, for *any* consistent formal system capable of formulating the truths of arithmetic, in the negative: given any such system, it is possible to formulate an arithmetic sentence which can (intuitively) be seen to be true but cannot be proven by the system, and it is impossible for the system to prove a statement of its own consistency (unless it is in fact *inconsistent*).

Gödel's argument from these results to his "disjunctive conclusion" in the lecture is relatively straightforward. The first incompleteness theorem shows that, for any formal system of the specified sort, it is possible to generate a particular sentence which we can "see" to be true (on the assumption of the system's consistency) but which the system itself cannot prove.[5] Mathematics is thus, from the perspective of any specific formal system, "inexhaustible" in the sense that no such formal system will ever capture *all* the actual mathematical truths. Of course, given any such system and its unprovable truth, it is possible to specify a *new* system in which that truth is provable; but then the new system will have its own unprovable Gödel sentence, and so on. The question now arises whether or not there is some formal system which can prove *all* the statements that *we* can successively see to be true in this intuitional way. If *not*, then human mathematical cognition, in perceiving the truth of the successive Gödel sentences, essentially *exceeds* the capacities of all formal systems, and mechanism (the claim that human mathematical coginition is, or is capturable by, a formal system) is false; this is the first alternative of Gödel's disjunction. If *so*, however, then there *is* some formal system that captures the capacities of human mathematical thought. It remains, however, that there will be statements that are undecidable for this system, including the statement of *its* consistency, which is itself simply an arithmetical statement. Thus it is impossible, on this alternative, simultaneously to identify the underlying principles on which actual mathematical cognition is based and to claim that these principles are both consistent and capable of deciding all mathematical problems. In this case there are thus classes of problems that cannot be solved by any formal method we can show to be consistent *or* by any application of our powers of mathematical cognition themselves; there are well-defined problems which will remain unsolvable, now and for all time.

We can further specify the underlying issue, and move closer to discerning its deep philosophical significance, by noting that, by Gödel's second theorem, the undecidable Gödel sentence for each system is equivalent (even within the system) to a statement, within that system, of its own consistency. As Gödel emphasizes, it is (given classical assumptions) an implication of the *correctness* of any system of axioms that we might adopt for the purposes of arithmetic demonstration that the system be consistent; but then it is an implication of the second incompleteness theorem that if we are in fact

[5] I here state the first theorem, roughly and intuitively, appealing to a notion of "truth" that is in some ways problematic. For discussion of the issues involved in the difference between this and other, less potentially problematic statements, see Livingston (2012), chapter 6.

using a specific (and consistent) formal system to derive all the mathematical truths (that we know) we could not know that we are. For if we could know this, i.e. if we could know the truth of the assertion of the consistency of the system, we would thereby know a mathematical truth that cannot be derived from that system. Accordingly, as Gödel says, it is

> …impossible that someone should set up a certain well defined system of axioms and rules and consistently make the following assertion about it: All of the axioms and rules I perceive (with mathematical certitude) to be correct and moreover I believe they contain all of mathematics.[6]

Thus if a system is (knowably) consistent it is, by that token, and *demonstrably* incomplete; if it is complete, we cannot know it to be consistent (and hence we cannot know it to be correct). Accordingly, on the assumption that we are in fact using a finite procedure to demonstrate mathematical truths, the assumption of the consistency of the system we are actually using is shown to be *essentially* unsecurable in any way that is itself consistent with our (in fact) using (only) that system at all.

Again, by considering the question of the axiomatization of mathematics, we can see how the issue is connected to the problem of the accessibility of the infinite, and the higher levels of infinity. Specifically, in order to axiomatize arithmetic set-theoretically without contradiction, it is necessary to introduce axioms in a step-by-step manner, and in fact, as Gödel suggests, this process can be continued infinitely: thus

> Instead of ending up with a finite number of axioms, as in geometry, one is faced with an infinite series of axioms, which can be extended further and further, without any end being visible and, apparently, without any possibility of comprising all these axioms in a finite rule producing them.[7]

The successive introduction of the various levels of axioms corresponds to the axiomatization of sets of various order types; in each case the introduction of a new level of axioms corresponds to the assumption of the existence of a set formed as the *limit* of the iteration of a well-defined operation.

But each axiom "entails the solution of certain Diophantine problems, which had been undecidable on the basis of the preceding axioms;" in particular, according to a result that Gödel had achieved in the 1930s, the consistency statement for any given system of axioms can be shown to be equivalent to a statement asserting the existence of integral solutions for a particular polynomial.[8] Since consistency is undecidable within the system itself, so is the problem of the truth-value of the statement concerned, but it becomes decided in a stronger system which adds, as a new axiom, a statement of the former system's consistency (or something equivalent to this).[9] But since the problem of the truth of the

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statement about the solutions to a polynomial is itself simply a number-theoretical problem, it follows that each particular system, if it *is* consistent, cannot solve some mathematical problem; and that *if* human cognition is equivalent to some *particular* system then there is some problem of this form (equivalent to the statement of its own consistency) that *it* cannot solve either. This is then an "absolutely undecidable" problem. If, however, there is no formal system to which human cognition is equivalent, then for *any* specified machine the mind can prove a statement which that machine cannot, and accordingly "the human mind … infinitely surpasses the powers of any finite machine."[10]

Gödel's argument in the 1951 article also turns centrally on Turing's demonstration in 19—of the unsolvability of the "Halting Problem" first suggested by Hilbert. The halting problem is the problem of finding a general procedure for determining whether any given algorithm (or Turing machine) will eventually halt, given any particular input, or will run on forever. As Turing demonstrated, there can be no such algorithm, since, supposing for *reductio* that one exists, it would then be possible to specify a Turing machine which halts if and only if it does not. The result implies not only (as Turing points out) the existence of real numbers whose decimal expansion is *uncomputable* (in the sense that there is no finitely statable procedure for determining the digits of the expansion) but also that first-order logic and stronger formal theories are *undecidable* in the sense that there is no finite *decision procedure* capable of determining, of any given formula, whether it is a theorem of the system or not.[11] As I shall suggest in the next several chapters, this systematic undecidability of formal, axiomatic systems has significant consequences for the "ontological" theory of the finite, the infinite, and their relationship to thought and practice. In particular, if the essential undecidability that Turing demonstrates henceforth marks a formally demonstrable limit to the effectiveness of formal procedures, this transforms in a basic way the question of the *accessibility* of the infinite to "finite" thought. If there is (as Turing's result demonstrates) no procedural means to decide the following of a given formula from a given formal system, and if (as Gödel's result demonstrates), for any such system (of enough strength to express arithmetic) there will be sentences that cannot be proven or refuted by any systematic means, then it is no longer possible *in general* to consider the truth of sentences to be decidable by any finite, procedural means. Thought meta-formally in a broader philosophical context and relevantly to the problematic relationship of thought to being in itself, this suggests, as I shall argue in more detail, the essential limitation of the faculties, capabilities or capacities of a *representing subject* with respect to the inexhaustibility of the infinite-Real.[12] This further suggests, as I shall argue in more detail over the next

[11] Undecidability in this sense – that of the undecidability of systems -- should be distinguished from the necessary existence, demonstrated by Gödel himself, of *sentences* in any system strong enough to capture arithmetic which are "undecidable" in the sense that the system cannot prove either the sentence or its negation.

[12]The issue can, again, be connected to that of the status of the most famous unsolved (and, as we now know, unsolvable) problem of set theory, Cantor's problem of the size of the continuum. From the work of Gödel himself in 1939 and Cohen in 1962-63, we now know that the continuum hypothesis (CH), which holds that the size of the continuum is the same as that of the first non-countable ordinal, cannot be demonstrated or refuted on the basis of the standard ZF axioms of set theory. Gödel himself thought, for a time at least, that the status of the continuum hypothesis might be resolved by the addition of one or more *new* axioms, in particular new axioms affirming the existence of certain "large" cardinals. If we were able intuitively to establish or otherwise have

several chapters, an essentially and formally demonstrable limit to the concept of the representing subject that begins in its modern form with Descartes and continues in Leibniz, Kant, Hegel, and German Idealism. Such a subject attains specific access to the infinite only by means of methodically specified rules or procedures applicable to the finitude of a kind of being constitutively limited in space and time, while on the other hand the specific sense of the infinite as such is thought, within this conception, as the infinite and unformalizable excess of a divine-Absolute, thus inaccessible as such to human cognition. This twofold conception, which receives its foremost expression in Kant as the dualism of the finitely constituted subject of faculties which stands under the necessity of schematizing the deliverances of empirical affection under the categories and the divine intellect capable an immediately creative intuition, is itself overcome in a twofold way by the complex of results that runs from Cantor to Turing, which demonstrate on the one hand the actual accessibility of the mathematically infinite to formal thought and, on the other, the inherent limitation of finite procedures in attaining to it. Besides thereby pointing to the formal and historical limits of the modern philosophy of subjectivity, Turing's result and the related ones to which Gödel appeals also, as I shall argue, have important implications for the ontological character and totalizing scope of what is called "information technology" today. In particular, if Turing thus demonstrates the inherent limits of the effectivity of formal procedures at the very moment at which he constructs the first formal definition of the structural architecture of a general computing machine, his result can be read as pointing to an ultimately inherent *ineffectivity* that thus accompanies the formalization of procedures and the imposition of "abstract" rule-based forms of reasoning and practice as their generally obscured but nevertheless structurally necessary underside. It is in terms of this specific structure of ineffectivity, as I shall argue in more detail, that the possibility of anything like a "reversal" or "overcoming" of the "metaphysical" essence of contemporary technology and its claim to global dominance can today be thought.

II

The two options left open by Gödel's disjunctive conclusion correspond directly to the two post-Cantorian orientations of thought, or positions on the relation between thought and being, that I called in *The Politics of Logic* the "generic" and "paradoxico-critical" orientations. On the first of Gödel's disjunctive options, the power of the human mind to grasp or otherwise comprehend truths beyond the power of any finite system effectively to demonstrate witnesses an essential *incompleteness* of any finitely determined cognition and a correlative capacity on the part of human thought, rigorously

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insight into the truth of some such axiom capable of resolving the status of the CH, this might provide evidence for the first horn of Gödel's disjunction, on which the power of the human mind to have insight into evident axioms true of mathematical reality essentially exceeds the capacities of axiom systems such as ZF. However, although the program of investigating the implications of such additional axioms continues actively today, none of the axioms that have so far been considered actually suffice to establish the truth of the CH, and none of them appear in any direct way "intuitively" motivated. Thus, the results of the inquiry so far might rather reasonably be taken to support the second horn of Gödel's disjunction, on which there are simply unsolvable problems; indeed, it might well be thought that the problem of the CH is one such, and that its unsolvability bears witness to an essential ontological feature of indeterminacy or undecidability characteristic of the universe of sets itself.[12]

following out the consequences of the mandate of consistency, to traverse by means of a "generic" procedure the infinite consequences of truths essentially beyond the reach of any such finite determination. On the second of the options, the essential undecidability of any such system witnesses, rather, the necessary *indemonstrability* of the *consistency* of any procedural means available to the human subject in its pursuit of truth, and thereby to the necessary existence of mathematical problems that are absolutely unsolvable by any specifiable epistemic powers of this subject, no matter how great. Both orientations, as I argued in the book, as well as the necessity of the (possibly non-exclusive) decision between them, result directly from working through the consequences of the systematic availability of the *infinite* to mathematical thought, as accomplished most directly through Cantor's set theory and its conception of the hierarchy of transfinite cardinals. More broadly, as I argued in the book, what is most decisive for the question of the orientations available to thought today is the consequences of the interlinked sequence of metamathematical and metalogical reflection running from Cantor, through Gödel's incompleteness theorems, up to Cohen's demonstration of the independence of the Continuum Hypothesis from the axioms of ZF set theory; it is thus not surprising that Gödel's own "philosophical remarks" about the implications of his own results should replicate the general disjunction in a clear and specific form.

The two "post-Cantorian" orientations are to be distinguished from the two "pre-Cantorian" orientations of *onto-theology* and *constructivism.* As I suggest in the book, each of the four orientations can be identified on the basis of the specific relationship it envisions between thought and being in connection with the idea of the *totality* of *what is* (in Heidegger's jargon, "Seiendes") as such. Both the onto-theological and constructivist orientations can be distinguished from the "post-Cantorian" ones in that they presuppose, though on different grounds, that this totality exists unproblematically in that it is possible for it to be both complete and consistent. In particular, whereas the onto-theological orientation assumes an infinite or transcendent consistent totality that is in itself never completely accessible to "finite" human cognition, the constructivist orientation constitutively involves the assumption of a knowable and consistent totality (for instance of what can be experienced or what can be referred to by means of a particular language) that is regulated, limited, or constructed by subjective procedures, activities, or forms. Because both of the "pre-Cantorian" orientations develop their respective conceptions of the thinkability of being in terms of the thought of a consistent and complete totality, they both amount to primarily *ontic* orientations toward what they figure as the totality of (what Heidegger would call) present-at-hand (*Vorhanden*) beings. Whereas onto-theology, understanding this totality as a complete and consistent whole quite independent of the capacities or activities of subjects or thinkers, captures most forms of realism that have been articulated in the metaphysical tradition, constructivism, in lodging the combination of consistency and completeness in the constitutive activities or abilities of a subject of experience, dialectical self-recognition, or linguistic institution, encompasses most forms of subjective, transcendental, speculative and linguistic idealism (or anti-realism). If, however, the set-theoretical and semantic paradoxes already indicate the untenability of the conjunction of consistency and completeness that both "pre-Cantorian" orientations assume, then the orientation of thought toward being can subsequently only be thought, as I have argued in the book, in one of the two "post-Cantorian" ways: either in terms of the generic orientation, which preserves consistency while sacrificing completeness (thus maintaining the consistency of rules of inference while sacrificing the existence of a total world to which they apply), or the paradoxico-critical orientation, which develops the thought of the *constitutively inconsistent* whole. As we shall see, the impossibility of the joint ("pre-Cantorian") assumption of consistency and completeness receives further confirmation, and is put on a farther-reaching basis, in relation to the constitutive ideas of practice, method, and procedure, by means of Gödel's and Turing's undecidability results. These results, as we shall see, jointly bear witness to an essential undecidability at the limit of all possible procedures, methods or activities of human (or any) rationality, insofar as these procedures, methods and activities are determined by rules in any sense. The formally indicated limit of the powers of the thinking subject then becomes visible as the closure of the metaphysical assumption of complete and consistent totality, and thereby of the whole historical epoch that Heidegger understands as that of the "metaphysics of presence".

Because both of the "pre-Cantorian" orientations develop their understanding of the relationship between thinking and being on the basis of the presupposition of the completeness and consistency of the *ontic* totality, they are equally (and in parallel fashion) overcome by the thought of the ontological difference. As Heidegger recurrently emphasizes in his discussions of the metaphysical tradition, the various configurations and approaches of metaphysics, beginning with Parmenides, always have in view the ontic totality of present-at-hand beings and always think this totality on the basis of the assumption of its joint consistency and completeness. What Heidegger calls the history of "onto-theology" is, as we have seen (chapter 1 above), in fact identical with the history of this assumption, and so the ontological displacement that it undergoes through Heidegger's thinking of the ontological difference and its radicalized consequences also provides ontological terms for the transition from the "pre-Cantorian" to "post-Cantorian" orientations.

It is true that Heidegger does not distinguish sharply between the two versions that this assumption can take: the "onto-theological" (in our sense) version which situates the consistent totality in a divine intellect or cosmological unity beyond the powers of finite or human thought and the constructivist one that identifies positive existence as what is thinkable by the finite subject. This unclarity in Heidegger's own retrospective discussion is, doubtless, at the root of the contemporary interpretive tendency which, while grasping to some extent the terms and implications of Heidegger's critique of onto-theology, nevertheless assimilate to him some form of the thesis that assimilates the "accessibility" of being as such to the powers of a thinking subject. Heidegger then appears as a kind of anti-realist with respect to being as such, and it is not difficult to locate in his writing, particularly in *Being and Time*, the conception of a structural "transcendence" which, as in Kant, is then thought to underlie the constructive relationship of something like a "human" subject – albeit now the living and factical subject of "emdodiment" and practices rather than the "intellectual" or "worldless" subject of Descartes, Kant, and (it is supposed) Husserl -- to what is thinkable of being as such. The conception has the additional merit of conforming well with a prevalent (and essentially constructivist) conceit of contemporary belief, according to which, if there is no "ultimate" theological referent to hold together the totality of the world as an intellectually thinkable unity, such access to being as it is possible "for us" to have must instead be facilitated, in irreducibly pluralistic fashion, by the variety of bodies, languages, and situated cultures. As we shall see in this chapter and the next one, however, an explicit identification of the metalogical issues at stake in the four orientations, and in particular to the inherent and essential meta-formal realism essentially presupposed in *both* of the post-Cantorian ones, points to a very different conception of the ontological problematic that is already suggested by Heidegger in *Being and Time*, albeit only developed fully in the course of his radical critical encounter with Kant in *Kant and the Problem of Metaphysics* and his subsequent development of the "grounding" question of the truth of being in contrast to the "guiding question" of beings. On this conception of the ontological problematic, it deconstructs the idea of a constitutively essential basis for ontology in "human" thought, language, culture, embodiment and practices just as thoroughly, and on substantially the same basis, as it does the "theological" intuition of the transcendent Absolute. As such, this conception, as we shall see, substantially underwrites the possibility of a realist conception of the ontological difference and of the ontological structure of time.

Most of the discussion in the philosophical literature over the broader implications of Gödel's theorems so far has been directed toward the question of the truth or falsity of *mechanism*. This is the question whether the mathematical thought of an individual subject, or perhaps of the whole community of mathematicians, can "in fact" be captured by some formal system. Gödel himself, particularly in his later years, was, as is well known, a dedicated anti-mechanist, and sometimes referred to his incompleteness theorems as providing evidence against mechanism; more recently, philosophers such as Lucas and Penrose have followed Gödel in arguing for this conclusion. Gödel also sometimes suggested that the truth of the first disjunct of his disjunctive conclusion in the Gibbs lecture, on which mechanism is false, might be established by means of independent (perhaps partly empirical) considerations. Nevertheless, the recent literature witnesses a consensus that (as Gödel himself seems to affirm in the lecture) the only conclusion relevant to the mechanism debate that can really legitimately be drawn from the incompleteness results themselves is the disjunctive one: *either* mechanism is false, and the human mind (or the community of mathematicians) has access to mathematical truths that cannot be proven by any formal system *or* mechanism is true and there are well-specified problems that cannot be solved by any means whatsoever.

Additionally, there are some good reasons to think that the "hypothesis" of mechanism cannot in fact be specified clearly or uniquely enough to use the incompleteness theorems to establish anything about its truth or falsity at all. Thus, for instance, in a recent very comprehensive review of discussion about Gödel and mechanism, Stuart Shapiro concludes that "there is no plausible mechanist thesis on offer that is sufficiently precise to be undermined by the incompleteness theorems."[13] One reason for this is that any proposal to treat the cognition of a subject, or human mathematical cognition overall, as embodying a specific formal system will clearly involve a significant degree of idealization with respect to actual practice; actual mathematicians make mistakes, and any determination of which formal procedure they are "actually following" would thus require a motivated distinction between what counts as mistaken performance and what does not. Similarly, any determination of *what* class of performance is to count as evidencing the postulated formal system is bound to be somewhat arbitrary; do we consider, for example, the behavior of just the *best* mathematicians, or all who are formally trained in (some kind of) mathematics at all, or perhaps of everyone who is even (minimally) competent

[13] Shapiro (1998), p. 275.

in mathematics at all? Finally, even if these worries about the idealization of performance can be overcome, one might wonder whether there is any "well-defined" way to consider questions involving the totality of *all* formal systems, as we must in fact do if we are to consider the truth-value of *either* term of Gödel's disjunctive result.[14]

For all of these kinds of reasons, it seems that it is not possible to draw any unequivocal conclusions directly from Gödel's incompleteness theorems about the hypothesis of mechanism with respect to human mathematical capacities. Nevertheless, despite these worries relevant to mechanism and idealization, it is possible to see the upshot of Gödel's "disjunctive conclusion" in the lecture as bearing relevance, beyond the issue of mechanism as well as the confines of "philosophy of mathematics" narrowly construed, to somewhat different philosophical issues.[15] In particular, it points to a distinctive and non-standard, but comprehensive position of *realism*, what I shall call *meta-formal* realism.[16] For this realism, the decisive issue is not, primarily, that of the reality of "mathematical objects" or the possibility of understanding them as determinate independently of the routes of access to them (epistemic or otherwise) involved in the exercise of our human capacities. It is, rather, that both terms of Gödel's disjunction capture, in different ways, the structural point of contact *between* these capacities and what must, on *either* horn of the distinction, be understood as an infinite *thinkable* structure determined quite independently of anything that is, in itself, finite. Thus, each term of Gödel's disjunction reflects the necessity, given Gödel's theorems, that any specification of our relevant capacities involve their relation to a structural infinity about which we must be realist, i.e. which it is not possible to see as a mere production or creation of these capacities.

On the first alternative, this is obvious. If human mathematical thought can know the truth of statements about numbers which are beyond the capacity of *any* formal system to prove, then the epistemic objects of this knowledge are "realities" (i.e. truths) that also exceed any finitely determinable capacity of knowledge. It does not appear possible to take these truths as "creations" of the mind unless the mind is not only credited with *infinite* creative *capacities*, but understood as having actually already *created* all of a vastly infinite and in principle unlimitable domain. But on the second alternative, it is equally so. If there are well-specified mathematical problems that are not solvable by any means whatsoever, neither by any specifiable formal system nor by human cognition itself, then the reality of *these problems* must be thought of as a fact determined quite independently of our capacities to know it (or, indeed, to solve them).[17] On this alternative, we must thus acknowledge the existence of a reality of

[14] I am indebted to discussion with Reuben Hersh for this point.

[15] I thus follow Feferman (2006), p. 11 in considering that, even if there are problems with applying Gödel's reasoning directly to the question of mechanism, "…at an informal, non-mathematical, more every-day level, there is nevertheless something to the ideas involved [in his argument for the "disjunctive conclusion"] and something to the argument that we can and should take seriously."

[16] In *The Politics of Logic* (p. 291), I called this position simply "formal realism". I add the prefix 'meta-', here, to reflect that what is concerned is not primarily an attitude (e.g. a Platonist one) about the "reality" or "actual existence" of forms, but rather the implications of the *transit* of forms in relation to what is thinkable of the real, the transit that can, in view of Cantor's framework, be carried out beyond the finite.

[17] Gödel says this about the second term of the disjunction: "… the second alternative, where there exist absolutely undecidable mathematical propositions, seems to disprove the view, that mathematics (in any sense) is only our own creation…So this alternative seems to imply that mathematical objects and facts or at least *something* in them

forever irremediable problems whose very issue is the inherent undecidability that results from the impossibility of founding thought by means of an internal assurance of its consistency. In this way the implications of the mathematical availability of the infinite, on either horn of the disjunction, decompose the exhaustiveness of the situation underlying the question of realism and idealism in its usual sense: that is, the question of the relationship of a presumptively finite thought to its presumptively finite object.

The actual underlying reason for the realism which appears forced upon us on either alternative is the phenomenon Gödel describes as that of the *inexhaustibility* of mathematics, which results, as we have seen, from the possibility of considering, given any well-defined ordinal process, its infinite limit (or totality). On the first alternative, this inexhaustibility yields a structurally necessary *incompleteness* whereby each finite system by itself points toward a truth that it cannot prove but which is nonetheless, by this very token, accessible to human thought. On the second, it yields an equally necessary *undecidability* which leaves well-specified mathematical problems unsolveable by any means (finitely specified or not) by any means whatsoever.[18] The form of the relevant realism is, in each case, somewhat different: the orientation underlying the first disjunct corresponds, as I argued in *The Politics of Logic*, to a realism of *truth beyond sense*, a position that affirms the infinite existence of truths and the infinite genericity of our dynamic insight into them beyond any finitely specifiable language or its powers, while the realism of the second consists is a realism of sense beyond truth, affirming the existence of linguistically well-defined *problems* whose truth-value remains undecidable under the force of any powers of insight whatsoever. But in either case, reflective thought about human capacities must reckon with the consequences of their structurally necessary contact with an infinite and inexhaustible reality essentially lying beyond the finitist determination of the capacities of the human subject or the finitely specifiable powers of its thought. In this way, the consequences of Gödel's theorem, however we interpret them, engender a structurally necessary realism about the objects of these powers that is the strict consequence of the entry of the infinite into mathematical thought.

It would probably not be difficult to show that each of the controversies between varieties of "realism" and "idealism", signed by prominent names in the history of philosophy, unfolds in direct and demonstrable connection with varying conceptions of the infinite and its availability to thought; one could consider, for instance, the difference between Plato's late conception of the Idea as owing its genesis to the ongoing struggle between the principle of the One and that of the *apeiron dyas*, or the unlimited dyad, and Aristotles merely *potential* infinity; or the difference between Leibniz's harmoniously ordered infinite continuity of monadic powers, up to the divine itself, and Kant's

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exist objectively and independently of our mental acts and decisions, i.e. to say some form or other of Platonism or "Realism" as to the mathematical objects." (pp. 135-36).

[18] Feferman () has demonstrated that there is a kind of "completeness" of arithmetic truths that is obtainable by the transfinitely repeated application of so-called "reflection principles", each of which amounts to adopting as a new axiom for a new system certain assumptions about the consistency of an earlier system (or the truth of its results). By means of an appropriate transfinite procedure through these principles, it is indeed possible, as Feferman shows, to obtain the totality of arithmetic truths. However, this procedure is itself not specifiable in a recursively enumerable way, and so does not provide anything like a general effective procedure for determining arithmetic truth. See Shapiro () and Berto () for discussion.

determination of the infinite as thinkable only in the form of the infinitely deferred, regulative idea). Nevertheless, wherever the actual infinite *has* been thought philosophically prior to the twentieth century, it has been thought simply as a theological (or, more broadly, onto-theological) Absolute. The singular significance of the event of Cantor thus lies, as Badiou has emphasized, in its rendering a de-absolutized infinite accessible to non-theological thought, in making mathematics as the "science of the infinite" the possible site for a renewed rigorously formal thinking of the powers and limits of thought.

As Gödel immediately goes on to point out, the only position from which it appears possible (while accepting Gödel's assumptions about mathematical reasoning and the incompleteness theorems themselves) to resist the "disjunctive conclusion" is a *strictly finitist* one according to which "only particular propositions of the type 2+2=4 belong to mathematics proper…"[19] and no *general* judgments applying to an infinite number of cases are ever possible. This kind of position would indeed avoid the disjunctive conclusion, since there is no way to apply the incompleteness theorems themselves consistently with it. However, as Gödel points out, the strict finitist view is very implausible as a view of mathematical reasoning, since it ignores that "it is by exactly the same kind of evidence that we judge that 2+2=4 and that a+b=b+a for any two integers a,b"; and it would moreover appear to disallow the use of even such simple "concepts" as "+" (which "applies" to all integers). Outside these very severely limited finitistic point of view, on the other hand, it appears inevitable that the disjunctive conclusion will apply, and thus we will be forced to acknowledge the validity of one or both of its disjuncts.

It is thus that the inherent character of reasoning in mathematics invokes the infinite, and marks the consequences of its availability to thought. As is evident in Gödel's interpretation of the implications of his own metaformal results, this kind of realism draws on the rigorous consequences of the formal thought of the infinite, and thus cannot be sustained solely within a position of finitism.

The attitude I am calling "metaformal realism" might certainly be developed as a position within the philosophy of mathematics itself. Developed in this way, it would bear a resemblance to a "methodological" realism about mathematics, for example of the kind suggested by Maddy (2005), that characteristically looks to mathematical practice itself as the source for its "ontological" claims and assumptions. This kind of realism has the advantage that it does not entertain, or attempt to solve, "metaphysical" problems about the "existence" of mathematical objects, except insofar as these problems are formulable and resolvable, in a motivated way, within mathematical practice itself (here, including the kind of "metamathematics" or "metalogic" that Gödel uses to produce his incompleteness theorems).

It is important to distinguish this kind of attitude from "Platonism" as it is traditionally construed. In particular, as Badiou (1998) has argued, there is no need to invoke, even in service of a realist attitude that here takes the event of the infinite and the consequences of mathematical practice seriously, the "Platonistic" claim of the "real existence" of mathematical objects. As Badiou suggests, the "Platonist" attitude of object-invoking realism is in fact quite alien to Plato's own concerns; in particular, it relies upon a "distinction between internal and external, knowing subject and known 'object'" which is, as

[19] Gödel (1951), p. 135.

Badiou says, "utterly foreign" to Plato's own thought about thought and forms.[20] Plato's fundamental concern is not, as Badiou argues, at all with the question of the 'independent existence' of mathematical objects, but rather with the 'Idea' as the name for something that is, for Plato, "always already there and would remain unthinkable were one not able to 'activate' it in thought."[21] Similarly, this is, as Badiou emphasizes, not an attitude of accepting or *believing in* the existence of sets or classes corresponding to well-defined monadic predicates, but rather one of maintaining, quite to the contrary, that what correlates to a well-defined concept may well be "empty or inconsistent"; it is thus a *metalogical* inquiry into the structure of forms for which, as Badiou emphasizes, "the undecidable constitutes a crucial category" and in fact becomes the central "reason behind the aporetic style of the [Platonic] dialogues," wherein thought constantly proceeds through forms to their own inherent points of dissolution or impasse. Whether or not we follow Badiou in his desire to redeem for this attitude the name of "Platonism," against its standard, ontological mis-approriation, what is most important to note is that what is involved here is thus not any direct attitude of realism toward objects of any kind but rather only a philosophical reflection of the internal consequences of the meta-formal inquiry into forms and their limits, including the open dialectic of finite and infinite thought.

Because this attitude, along with Plato himself, accords mathematical experience a certain privilege as, precisely, a non-empirical experience of forms, the realism suggested by it *can* be worked out, as I have said, as a position within the "philosophy of mathematics" itself. But it seems to me that the kind of realism exhibited here can also find fruitful application more broadly, to domains other than simply that of mathematics. For as I argued in *The Politics of Logic*, the consequences of formalism and formalization in their contemporary practical and theoretical development are by no means limited to mathematics, but extend to a broad range of phenomena and many aspects of contemporary social and political life. As a leading example of this (though there are certainly others) one might consider the pervasiveness of informational and computational technologies and the forms of abstract social organization they make possible, themselves grounded in the technology of the computing machine which was directly made possible by the development of the implications of the concept of a formal system in thinkers such as Hilbert, von Neumann and Turing. If this and many other developments of twentieth century praxis and organization are indeed, as I argued there, intimately linked to the project of formalization in its various dimensions, then a realism that is, as I have suggested, itself directly linked to the aporeatic result of this project's development may be singularly appropriate to contemporary critical and reflective thought.

Here, as I argued in the book, the relevance of leading developments in mathematics and metamathematics is thus not limited to the "philosophy of mathematics" narrowly construed, but extends to the broader impications of the ongoing project of formalization itself. If, accordingly, the *metaformal realism* I am recommending here arises in an intrinsic way from the structure of forms in their capture of life, then a rigorous understanding of the relationship of thought to being may today require such a position, which takes account of the implications of the dimensions of the infinite as they occur at the horizon of our contemporary understanding of ourselves and the world. The specific

[20] Badiou (1998), p. 49.
[21] Badiou (1998), p. 49.

relevance of mathematics and metamathematics, in this connection, does not lie in the identification of a particular realm or region of entities, but rather in the way that mathematics, *as* the "science of the infinite", possesses the ability to capture and schematize the constitutively "infinite" dimension of form itself. As I argued in the book, this infinite dimension of forms is a constitutive part of the thinking of form, even when it is dissimulated or foreclosed, ever since Plato, and is inherently involved, as well, in every contemporary project of the analysis of logical form or the discernment of the formal determinants of contemporary life and practices. This twentieth-century inquiry into formalism has, as I argued in the book, many interacting dimensions, including (but not limited to) the philosophical inquiries, both "analytic" and "continental," which in the twentieth century interrogate the structures of language as essential guidelines to their inquiry into forms of life. As such, its results capture the most important implications of contemporary reflective inquiry for the constitutive idea of the rational human subject or agent of capacities and thought.

In particular, as is clear in relation to Gödel's development of his results, this metaformal realism, with its constitutive conception of the powers of thought in relation to a real determined as infinite, marks the unavailability of any traditional opposition between the finitude of the human subject and a transcendent matter thought under the heading of the absolute. If, on the contrary, thought is capable, in its capacity for formalization, of rigorously conceiving an infinite-real to which it is immediately adequate (whether this capacity be thought as itself infinite, or as grounded in the finite systematics that comprise a formal system), then it is no longer possible to oppose an attitude of realism (in the traditional sense) to one of idealism according to the different positions taken on thought's capacity to know its object in itself.

Metaformal realism, as I have discussed it here, is an essentially *disjunctive* position, split between affirming the consequences of two quite distinct and mutually incommensurable orientations of post-Cantorian thought, the generic and the paradoxico-critical. As we have seen, Gödel's own disjunctive result witnesses just this disjunction with respect to the powers of human thought in relation to a mathematical reality which the constitutive thought of the infinite determines as the inexhaustible-real: this is, in Gödel's terms, the essential distinction between, on one hand, the assumption of an inherent and transcendent power of human thought to bear witness to consistency by exceeding the powers of any finitely specifiable system of rules, and on the other, an inexhaustible inscription of the undecidable as such, including the undecidability of consistency itself, in the very structure of mathematical reality. Because he was a committed anti-mechanist, Gödel favored the first disjunct (on which the human mind is non-mechanical) and sometimes argued against the tenability of the second on independent grounds, holding both that it ignores the inherent capacity of the human mind to innovate with respect to its guiding axioms and principles and that the existence of absolutely unsolvable problems is untenable since it would imply that "it would mean that human reason is utterly irrational by asking questions it cannot answer, while asserting emphatically that only reason can answer them."[22]

[22] Wang (1974), p. 324, discussed in Feferman (2006), p. 12. The former point, about the non-static nature of the mind, is made against Turing's own position in Gödel (1972), remark 3, p. 306.

However, once we have acknowledged the implications of the availability of the infinite to mathematical thought and made the general decision for meta-formal realism at all, there are some important senses in which the second disjunct, corresponding to the orientation of paradoxico-criticism, is not only not excluded but also enjoys advantages over the choice for the first disjunct (which Gödel himself preferred). In particular, besides being more obviously compatible with materialism because not in any way at odds with mechanism, the paradoxico-critical outlook makes it possible to preserve an outlook and practice that continues the classical orientation of *criticism* with respect to the capacities and practices of the human subject, in the altered conditions post-Cantorian thought. To gain a sense of these ongoing critical implications, one might usefully juxtapose Gödel's remark about reason posing problems that it cannot solve with the infamous opening lines of Kant's first *Critique*:

> Human reason has this peculiar fate that in one species of its knowledge it is burdened by questions which, as prescribed by the very nature of reason itself, it is not able to ignore, but which, as transcending all its powers, it is also not able to answer.

Kant, of course, was a transcendental idealist; and within the fourfold framework of orientations of thought I developed in *The Politics of Logic*, Kant's thought remains a paradigm of the pre-Cantorian constructivist (or criteriological) orientation, which is defined by its attempt to assay the boundaries of knowledge from the exterior position of a limit-drawing project committed to saving jointly the ideas of completeness and consistency. In the post-Cantorian context, it is no longer possible to save these ideas jointly, and so the constructivist orientation and its associated kind of idealism are both rendered untenable. But by making the paradoxico-critical decision for the combination of a rigorous inquiry into totality with the implication of irreducible paradox at the boundaries, it is possible to maintain the properly critical register of Kant's thought of reflective reason in its ongoing dialectic with itself, and to situate this thought within, as I have argued, a rigorously *realist* position with respect to the relation of thought and being itself. To do so is to transpose the ultimate ground for the development of such a dialectic (now thought more in a properly Platonic rather than a Kantian or Hegelian sense) decisively away from the (pre-Cantorian) Kantian oppositional figure of opposition between the finitude of sensory affection and the absolute-infinite divine intellect capable of intellectual intuition, and to reinvent the possiblities of critique on the ontological real ground of the objective undecidability of problems that are problems for (finite or infinite) thought *in itself*, given to it at the point of its very contact with the real of being as such.

What, then, are some of the concrete effects of this transposition for contemporary reflective and critical thought? As I argued in *The Politics of Logic,* most generally, the necessity, in a post-Cantorian context, of the forced choice between inconsistent completeness and incomplete consistency indicates, as is confirmed by Gödel's development of the philosophical consequences of his own results, that it is impossible *by* finite, procedural means to confirm rigorously the *consistency* of the finitely specifiable procedures of our social-political, practical, and technological worlds. This suggests, as I argued at more length in the book, that it is impossible by finite means to ensure the *effectivity* of our practices, or procedurally to found whatever faith we may maintain in their ongoing extensibility and capability of continuation. This faith, if it is to be founded at all, must be founded in an essentially infinite capacity of insight and fidelity, bordering on the mystical, to a Real matter of consistency with respect to our own practices that can itself never be guaranteed by any replicable or mechanical procedure; *or* it must be ceaselessly decomposed and deconstructed at the point of the inherent realism of the problematic and undecidable that is necessarily introduced if this faith cannot be assured at all. Such are the consequences, as I have argued in *The Politics of Logic*, of the transformative event of the development of formalization in the light of the accessibility of the mathematical infinite that characterizes our time; and such are the stakes, as I have tried to confirm here, of the metaformal realism that this event rigorously motivates and demands.

The metaformal realism thus indicated has several further distinctive features, which I briefly adumbrate:

1. Metaformal realism is neither a "metaphysical realism" nor an "empirical realism."." In particular, because it is grounded solely in an *internal* experience of the progress of forms to the infinite, it avoids any need to posit an empirical or transcendent referent beyond the effectiveness of forms and formalization and does not ground its realism in any such referent. Because of the way it turns on the entry of the infinite into mathematical thought, it does not require that one assure oneself of the existence of a world "in itself" and independent of thought. It is thus completely distinct from any realism of a "mind-independence" variety, which always requires a problematic doctrine of the bounding of thought in relation to its empirical objects. It also does not require, and does not encourage, the possibility of a "view from nowhere" or a "single unique description of reality."
   Rather, we have here a rigorous internal development of the limitology of thought from within thought itself, a development of "thought thinking itself" which is nevertheless not 'dialectical' and does not attest, either, to the power of thought consistently to appropriate *everything* within itself. For all of these reasons, metaformal realism does not involve the difficult metaphysical and epistemological questions (how is it possible to know or have access to a "thing in itself"? What is the status of the "world independent of the mind"?) which recurrently appear to make forms of "metaphysical realism" untenable and have often been taken to motivate a contrasting position of idealism (or pragmatism, or 'internal realism,' etc.)
2. Metaformal realism is thus a *reflective*, not a 'speculative' realism. It develops all of its consequences internally, from internal reflection on the limitology of thought and its inherent formal features. It thus has no need to posit an object of *speculation* simply external to this limitology or to engage in the uncertain investigation of the features of such an object. If it is, as I shall try to show, engaged in an inherent dialectic of thought with being, this dialectic is thus not a *speculative* dialectic of "determinate negation."[23]

[23] I refer here, in passing, to the distinction between "reflection" and "speculation" drawn by Hegel in the "Preface" to the *Phenomenology of Spirit*, para. 59. That I thus distinguish the post-Cantorian orientations of metaformal realism from Hegel's pre-Cantorian speculative dialectic should not exclude that metaformal realism, particularly in its paradoxico-critical variant, nevertheless exhibits a number of important parallels to aspects of Hegel's system, particularly in its treatment of the nature of contradiction *prior* to its dialectical sublation or resolution; for discussion of some of these relationships to Hegel, see *The Politics of Logic*, pp. 253-54.

3. Metaformal realism de-absolutizes the world as a transcendent object of thought. As I argued in *The Politics of Logic*, the twentieth-century inquiry into forms, pursued in its narrower aspect as the inquiry of "metamathematics" or "metalogic," has the consequence of consigning formal thought about the totality of the world (indeed, thought about totality in general) to an unavoidable disjunction, what I called there the "metalogical duality" between consistent incompleteness and inconsistent completeness, essentially the same alternatives involved in Gödel's 'disjunctive' conclusion. This means, as well, a basic diremption of any figure of thought that countenances a (complete *and* consistent) Absolute, and forces a choice between acknowledging the essential incompleteness of consistent thought or countenancing the existence of the totality of the world only under the heading of the reality of the inconsistent.

III

In contemporary philosophical discourse, no project has done more to illuminate the issue of realism and its underlying formal determinants than Michael Dummett's. Familiarly, in a series of articles and books beginning in 1963 with the article "Realism," Dummett has suggested that the dispute between realism and anti-realism with respect to a particular class of statements may be put as a dispute about whether or not to accept the principle of *bivalence* (i.e., the principle that each statement is determinately true or false) for statements in the class concerned.[24] Though this issue yields differing consequences in each domain considered, the acceptance of bivalence generally means the acceptance of the view that all statements in the relevant class have truth values determined in a way in principle independent of the means and methods used to verify them (or to recognize that their truth-conditions actually obtain when they, in fact, do so); the anti-realist, by contrast, generally rejects this view with respect to the relevant class. Dummett did not envisage that this comprehensive framework would or should support a single, *global* position of metaphysical "realism" or "anti-realism" with respect to all domains or the totality of the world; rather, his aim was to illuminate the different kinds of issues emerging from the traditional disputes of "realism" and "idealism" in differing domains by submitting them to a common, formal framework.[25] From the current perspective, however, it is just this aspect of formal illumination which is the most salutary feature of Dummett's approach. For by formally determining the issue of realism with respect to a given domain as one turning on the acceptance or nonacceptance of the (meta-)formal principle of bivalence with respect to *statements*, Dummett points toward a way of conceiving the issue that is, in principle, quite independent of any *ontological* conception of the "reality" or "ideality" of *objects* of the relevant sort. In particular, it is in this way that Dummett avoids the necessity to construe realism and anti-realism in *any* domain as involving simply differing attitudes toward the ontological status of its objects (for instance that they are "mind-independent" or that, by contrast, they are "constituted by the mind"). What this witnesses, *along with* what I have called meta-formal realism, is the possibility of a purely formal and reflective determination

[24] Dummett (1963); for some later reflections on the development of the framework and issues related to it, see Dummett (1978).
[25] Dummett (1978), pp. xxx-xxxii.

of the issue of realism that connects its stakes directly to those of the truth of claims, thereby instantly short-circuiting the laborious and endlessly renewable dialectic of the "actual relationship" of mind to world.

Dummett's framework is sometimes glossed in terms that suggest that, for him, the adoption of realism or anti-realism in any particular case turns primarily on our judgment about the (primarily epistemological) issue of whether a certain type of entities can be considered to be real in themselves, independently of our access to them or ability to possess evidence for their existence. But that this kind of formulation is, at best, highly misleading, both with respect to Dummett's own motivations and the actual merits of the framework he recommends, can be seen from the introductory formulation of the issue of realism and anti-realism in the original article "Realism" itself:

> For these reasons, I shall take as my preferred characterisation of a dispute between realists and anti-realists one which represents it as relating, not to a class of entities or a class of terms, but to a class of *statements*, which may be, e.g., statements about the physical world, statements about mental events, processes or states, mathematical statements, statements in the past tense, statements in the future tense, etc…[T]he realist holds that the meanings of statements of the disputed class are not directly tied to the kind of evidence for them that we can have, but consist in the manner of their determination as true or false by states of affairs whose existence is not dependent on our possession of evidence for them. The anti-realist insists, on the contrary, that the meanings of these statements are tied directly to what we count as evidence for them, in such a way that a statement of the disputed class, if true at all, can be true only in virtue of something of which we could know and which we should count as evidence for its truth. The dispute thus concerns the notion of truth appropriate for statements of the disputed class; and this means that it is a dispute concerning the kind of *meaning* which these statements have.[26]

There are two points here that bear important implications for the issue of how best to characterize realism and anti-realism. The first is that, on Dummett's formulation, it is an issue, not of the reference of terms or the existence of objects, but of the way in which the truth-values of statements are determined. The second, following from the first, is that the question of realism within a given domain is not directly an epistemological question about our knowledge of (or 'access to') entities, but rather a semantic question about the basis of the *meaning* of statements. As Dummett points out, both points are helpful in characterizing the real underlying issue and separating it from other issues that have become confused with it in the history of discussion of realist and idealist (or nominalist and universalist, etc.) positions. For example, in the traditional debate between phenomenalists and realists about material objects, which has sometimes been put as a debate about their "existence", Dummett argues that his framework allows the actual question of realism to be separated from what is in fact a conceptually different one, the question of *reductionism* (i.e. of whether 'material objects' can in fact be reduced to something like sense-data). Somewhat similarly, with respect to mathematics, concentrating on the question of the reference of terms tends, Dummett suggests, to "deflect the dispute from what it

[28] Dummett (1973).

is really concerned with"; in particular, "the issue concerning platonism relates, not to the existence of mathematical objects, but to the objectivity of mathematical statements."[27] Here again, a framework primarily directed toward the question of the meaning of statements is more useful than one concerned primarily with questions of the existence of objects. This is, at least in part, because in mathematics (as opposed to some other cases) it is generally implausible to suppose we can have "access" to the relevant "objects" independently of a recognized procedure (i.e. a calculation or a proof) for establishing the truth of statements about them; and on the other, that such a procedure is also generally taken to be *sufficient* for whatever access to mathematical objectivity we can enjoy.

Although this kind of consideration finds application quite generally, it is certainly no accident that the historical dispute which forms the basic model for Dummett's formal framework itself is the dispute between formalists and intuitionists about the foundations of mathematics in the 1920s and early 1930s. Partisans of the two positions reached deeply opposed conclusions about the nature of reasoning about the infinite, but for both positions the idea of a *finite (i.e., finitely specifiable) procedure* or process of demonstration plays a central role. In particular, whereas the formalist position allows the axioms and rules of a formal system to extended classically, by means of such a procedure, to arbitrarily extended reasoning about the infinite *provided* that the system can be shown to be consistent, intutionism generally restricts the positive results of mathematics about the infinite to what can be shown by means of a finite, constructivist procedure of proof.

In the 1973 article "The Philosophical Basis of Intuitionistic Logic," Dummett considers the question of what rationale might reasonably serve as a basis for replacing classical logic with intuitionistic logic in mathematical reasoning (hence, in his framework, for replacing realism with anti-realism).[28] As Dummett emphasizes here, the decision between realism and anti-realism depends ultimately on our conception of how *sense* is "provided" for mathematical statements, and in particular whether we can conceive of these statements as having sense quite independently of our means of recognizing a verification of them. It is thus, ultimately, as Dummett sees it, general issues about the *capacities* or *practices* that we learn in learning a language and deploy in speaking one that determine, given his framework, equally general issues about whether realism or anti-realism is better justified in any given domain. As in the earlier article "Realism," Dummett here emphasizes that this primary issue is not an epistemic or ontological, but rather a semantic one. Thus, "Any justification for adopting one logic rather than another as the logic for mathematics must turn on questions of *meaning*"; and again, "it would be impossible to construe such a justification [i.e. for adopting classical or intuitionistic logic] which took meaning for granted, and represented the question as turning on knowledge or certainty."[29] In fact, Dummett suggests, there are just two lines of argument that could plausibly be used to support the replacement. The first turns on the idea that "the meaning of a mathematical statement determines and is exclusively determined by its *use*"; beginning from this assumption, it is plausible to hold that any difference between two individuals in their understanding of mathematical symbolism would have to be manifest in observable differences of behavior or capacities. The second turns on considerations about

[28] Dummett (1973).
[28] Dummett (1973).
[29] Dummett (1973), p. 215.

learning, and in particular on the thought that what it is to learn mathematical reasoning is to learn how to *use* mathematical statements (i.e. when they are established, how to carry out procedures with respect to them, how to apply them in non-mathematical contexts, etc.).[30] On either assumption, it is then reasonable, Dummett suggests, to hold that *since* meaning is exhausted by use (in one way or the other) we cannot claim that a notion of truth, understood classically as imposing bivalence on all mathematical statements independently of the use we actually make of them, can any longer serve as the "central notion" for a characterization of the meanings of mathematical statements. In place of the classical notion of truth, Dummett suggests, we must substitute a notion grounded in the practices of which we have actually gained a mastery; in particular, we must replace the classical notion of truth with the claim that "a grasp of the meaning of a statement consists in a capacity to recognize a proof of it when one is presented to us."[31] (p. 225). This, in turn, allows the recognition that certain classical arguments and proof-procedures are unjustified from this perspective, and should accordingly be replaced with intuitionistic ones.

Dummett thus presents the best route to the adoption of intuitionistic logic in mathematics as motivated by considerations very different from those that motivated arguments to the same conclusions for classical intuitionist thinkers such as Brouwer and Heyting; in particular, as Dummett points out, whereas intuitionism was motivated for those thinkers primarily by the requirement that mathematical objects be present or given in subjective, private experience, Dummett's arguments turn on what is in some ways the exactly opposite idea, namely that of the mastery of a socially learned and publically evident intersubjective *practice*. In fact, Dummett suggests against the views of the early intuitionists, there is *no* plausible route from the view that mathematical entities such as natural numbers are "creations of human thought" to the application of intuitionistic logic, unless we are prepared to adopt a very severely restricted (and implausible) view of mathematical practice (including rejecting unbounded quantification over all numbers, etc.)[32] For this reason, Dummett suggests as well that there is no good reason to think that any successful argument for anti-realism in mathematics can turn on considerations bearing simply on the supposed ontological peculiarities of the mathematical domain; both of the reasonable arguments that one might make turn, instead, on considerations about the link between meaning and use which have nothing special to do with mathematics and would seem to be applicable much more broadly, to any number of classes of sentences "about" widely differing kinds of things.

By posing the issue of realism vs. anti-realism, not only in the mathematical case but more generally, as turning on the question of the provision of sense, Dummett shows that the question of realism in a particular domain is most intimately related, not to the question of the ontological status of, or our epistemological access to, its objects, but rather to the question of the coherence and range of the procedures by means of which the *meanings* of statements about the domain are learned and manifested. But this is none other than, again, the question of the way that the infinite becomes available on the basis of a finite procedure. For the intuitionist (and by analogy, the anti-realist more

[33]Feferman completeness; Wittgenstein's model-theoretic argument (Putnam, etc.)
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generally), it is possible to establish the existence of an object only if it can be shown to result from its actual construction in a finite number of steps or from a finite, constructivist proof (i.e. one that does not involve reasoning over arbitrarily complex infinite totalities); by contrast, for the formalist (realist), all that is needed is to show that it is possible to refer to the object without contradiction within a specified formal system. And it is just here, with regard to the specific question of what is involved in the learning and pursuit of a finite procedure, that the possibility of meta-formal reflection of the sort that I have portrayed Gödel as engaging in proves to be decisive. For Gödel's own incompleteness theorems, of course, result directly from a rigorous meta-formal consideration of the range and capacities of formal systems (in Hilbert's sense and related ones). In particular, Gödel's first theorem shows that for any such system, there will be a number-theoretical sentence that is beyond its capacity to prove or refute, and the second theorem shows that no such system can prove its own consistency (assuming that it *is* consistent). In this way Gödel's results render the formalist conception of finite procedures unsuitable for anyone who wishes to assert the realist position that the statements of number theory have determinate truth-values, independently of our ways of verifying them; but on the other hand, in invoking under the heading of the "inexhaustibility" of mathematics an essential reference to a reality that marks the point of impasse of any given finite procedure, Gödel's argument shows the intuitionist strictures to be untenable as well.

Just as Gödel's theorems themselves thus overcome the debate between intuitionism and formalism, narrowly construed, by conceptually fixing and reflecting upon the contours of a central concept (that of a finite procedure) commonly appealed to by both, the meta-formal realism I have discussed as suggested by Gödel's argument provides a new basis for critically interrogating the central concept of a *rule of use*, as it figures in both "realist" and "anti-realist" conceptions of the structure of language. In particular, as I argued in more detail in *The Politics of Logic*, we may take considerations analogous in some ways to those which establish Gödel's second incompleteness theorem to demonstrate, the incapacity of a finitely specifiable system of such rules to establish its *own* consistency. It is then apparently possible to draw, with respect to our actual practices and institutions of linguistic use, a conclusion directly analogous to that drawn by Gödel with respect to mathematical reasoning specifically: namely that *either* the consistency of our regular practices can only be known, and assured, by a deliverance of an essentially irregular insight that essentially cannot be subsumed within them or determined by them insofar as they can be captured by rules; *or* it cannot be known at all and thus can *only* be treated as a perpetually deferred problem. On either assumption, the claim of consistency is shown to be, from the perspective of the regular provision of sense, the point of an impossible-Real that always escapes, drawing along with it any possibility of an internal systematic confirmation of the infinite noncontradictory extensibility of the rule to ever-new cases. It is in this way, as I have argued, that the phenomenon that Gödel calls the "inexhaustibility of mathematics" points toward a metaformally justified realism of the impossible-Real, correlative to what we may describe as our essential openness toward the infinite and based in metaformal reflection about the limits and transit of forms. In so doing, it unhinges any possible claim of the humanistically conceived "finite" subject finally to ground itself, or to secure by its own means the ultimate sense of its language and life.

What, then, of Dummett's own arguments for anti-realism in various domains? The general form of this argument is the one we have already seen with respect to mathematics. It turns most centrally, as we have seen, on the question of how sense is "provided" for the range of statements characteristic of the entities of a given domain. In particular, on the assumptions about the basis of sense that Dummett attributes to the late Wittgenstein (correctly or incorrectly), sense must be provided or established for any range of sentences by means of the establishment and learning of a (public, intersubjective) *practice*. On Dummett's various arguments, this makes it incoherent, in a variety of domains, to suppose that sense could have been provided or determined independently of procedures for verifying truth in those domains; according to Dummett, it is for this reason, for instance, that one must be anti-realist about descriptions of "private" experience, and these considerations at least suggest anti-realism about the past (though as Dummett admits, matters are more complex here owing to the internal complexity of the notion of the (current) verification of (past) events itself). The general argument goes through, as we have seen, on the assumptions that i) sense is provided by means of a practice which essentially *involves* laying down various well-defined procedures of verification and ii) this provision of sense is *not intelligible except by means* of the specification and establishment of these procedures. However, these assumptions are at least contestable in a context where (as I have argued with respect to the kind of metaformal reasoning Gödel applies) sense appears to be "provided" through finite instances of teaching and learning, but in a way that essentially outstrips any description of them as finite *procedures*; and truth is demonstrated, not by any simple application of established verification procedures, but on a constitutive *reflection* on their scope and limits that is itself *irreducible* to any antecedently given procedure. More generally, since Dummett's argument turns on the thought that sense, if it is to be determinate, must first be "provided" by the human activity of instituting such procedures, it can be resisted where we have good independent reason to consider sense to be "given" in a way that essentially outstrips these procedures. Such an independent reason is provided, as I have argued, by the metaformal reflection that underlies Gödel's results and thereby demonstrates what he calls the "inexhaustibility" of mathematics, and more generally by the problematic accessibility of the infinite and transfinite to thought that is broadly witnessed in the results of Cantor, Gödel and Turing.

More narrowly, Dummett has at least sometimes suggested an argument from Gödelian incompleteness itself to anti-realism about mathematics. The argument is that, since as Gödel shows there are undecidable sentences for every consistent formal system capable of expressing arithmetic, there must be sentences, for any such system, that cannot be verified as true or false by means of its proof procedures. Assuming that truth an falsehood are intelligible only in terms of intra-systematic proof, there must then be, for any such system, sentences that are neither true nor false, and it may be thought to follow as a corollary that an intuitionist logic must therefore be adopted to treat them. This conclusion is, of course, very different from the disjunctive one Gödel himself draws from his incompleteness results, whose two alternatives must both be understood, as we have seen, as robustly realist. In fact, to argue *from* undecidability to intuitionist logic in the way that Dummett at least sometimes suggests is in a certain way to ignore the deeper underlying reasons *for* undecidability. For the argument involves *assuming* that proof can only be intelligible as an intra-systematic notion; whereas Gödel's first theorem itself depends on "verifying" the undecidability of the Gödel sentence for a particular system from an essentially *extra*-systematic perspective. And the possibility of this verification bears witness to that of a kind of "insight" into reality that is not simply the outcome of a proof procedure in any particular formal system. If we can assume that the Gödel sentence for a particular system, construed simply as an arithmetical sentence, has reference to the "actual" natural numbers, it is natural to put this "insight" as the insight into a truth about them that the system in question essentially cannot prove (if it is consistent). But in fact to appreciate the more general possibility of a genuine extra-systematic insight into the Real here, it is not necessary to conclude, as Gödel himself most often does, that the Gödel sentence for a particular system is an arithmetical truth in this sense. Even on the other horn of the disjunctive conclusion, where what is demonstrated is not a particular arithmetic truth but simply the undecidability of a sentence for a particular system, *this* undecidability is still demonstrated as a positive fact about this system (again on the assumption that it is consistent), and it is also possible to draw the more general conclusion that *every* formal system will evince some such sentence. What is gained with this insight, even if it is not a successive insight into the "truths" about natural numbers, conceived "Platonistically" as existences in and of themselves, is nevertheless a general meta-formal insight into the real limitations of all regular formal systems as such insofar as they are capable of touching on truth. On this horn of the disjunction, it thereby points, as I have argued, to an irreducible insistence of problems unsolvable by any procedural means, an insistence that must itself be considered an irreducible mark of their reality.[33]

With respect to this problematic insistence of the Real at and beyond the limits of finite procedures, which shows up, as I have argued, only when the very idea of a finite procedure is subjected to critical and meta-formal reflection, along with the constitutive ideas of the finite and of capacities that underlie it, the question that Dummett characteristically asks about the relationship between the instituting or specifying of "use" and that of procedures of verification is thus not the most telling one. Rather, the real question is prior to this: it is the question of how *any* institution – or its communication in teaching or learning – can suffice to determine an infinite totality of truths about objects in a certain domain to begin with. But this is just the question underlying Wittgenstein's "rule-following" considerations in the *Investigations*, and Dummett's failure to discern the tension between Wittgenstein's critical inquiry into the very idea of rule-following and the "official" conception of "meaning as use" marks a real limitation of his reading of him.[34] As Wittgenstein argues, it is evidently no answer to this question to hold that the infinite number of truths about objects in a particular domain are determined "all at once" by means of the inscription of a symbolic formula or an experiential intuition of its "meaning." To say that language, or the use of a word, is a "practice" or "institution" is thus not to say that it is determined or determinable, once and for all, by means of a finitely specifiable regular procedure specified or specifiable in advance; but rather that its form is shown in what we do from case to case of new applications. Specifiable "procedures" for verification are given along with this general form as it is lived, and as emerge from it by means of explicit formal reflection. But this does not mean that the understanding of the truth or falsity of claims that is shown in these instances and given in this form must be constrained by them in such a way that it is not possible to maintain their "realist" reference to

[33]Feferman completeness; Wittgenstein's model-theoretic argument (Putnam, etc.)

[34] This failure is doubtless responsible for Dummett's attribution to Wittgenstein, in a famous and critical review of the latter's *Remarks on the Philosophy of Mathematics*, of a "full-blooded" conventionalism on which the result of a new calculation must be spontaneously decided in each case.

things as they are in themselves and to the correlative determinacy of truth and falsehood.[35] Moreover, as the results of Gödel and Turing show, there in fact *must* be, given any formal procedure of verification of a certain type, "realist" truths that cannot be established thereby, whether of the actual truth-value of propositions, e.g. of the Gödel type, or about the irreducible insistence of *problems* that, though completely determinate in themselves, essentially evade *any* procedural/regular solution.

Viewed this way, the realism that is recommended on either horn of Gödel's disjunctive conclusion in fact has implications far beyond the domain of "mathematical" truth itself. In particular, a similarly motivated meta-formal realism is recommended wherever it is possible (or suggested on independent grounds) to be realist about *sense* itself. We obtain this realism as soon as we recognize that the *constitution* of sense in any particular domain is not simply the result of its *construction*, whether by means of the capacities or activities of a finite "human" subject or indeed by the institution of finitely specifiable regular procedures, rules, or norms of "intersubjective" practice. As we have seen, Dummett's framework has the salutary benefit of resituating the question of realism as a question of the determinacy of sense rather than as the old question of the constitution of objects in relation to our ways of knowing about them. But in applying it and especially in arguing for anti-realism, he tends to assume that sense *must* be "provided" by means of socially instituted practices, if it is to be provided at all. What meta-formal realism points to, by contrast, is the possibility of a realism about the "provision of sense" that separates it from any constructivism, whether of a subjectivist, social-pragmatist, or finitist kind. From the current perspective, the problem of this constitution is not distinct from the problem of the accessibility of the infinite to thought itself. For (linguistic) sense is in itself infinite, if only for the reason that knowing or understanding the sense of a single term involves, in principle, knowing how to apply it in an infinite number of cases. If, as I have argued, the complex of results running from Cantor, through Gödel, to Tarski, shows the irreducibility of this access to any finitely specifiable procedure, it also thus motivates a realism about sense and its givenness that outstrips any determination of this givenness in terms of (finitely specifiable) capacities, abilities, faculties or practices. Although, as we have seen, this does not by itself demand or establish realism about any particular domain of entities or referents, it is the appropriate meta-formal basis for an "ontologically" realist position about sense in its constitutive relation to the being of beings. For this reason, as we shall see over the next few chapters, it can also be the basis of a robust realism about *time* in its fundamental structure, in opposition to the "metaphysical" ultimately anthropological intuition, running from

[35] Something similar can be said, as well, about Wittgenstein's own complex and much disputed suggestions in the *RFM* about the Gödel sentence and the meaning and bearing of Gödel's proofs. In particular, though Wittgenstein is certainly dismissively critical of the thought that Gödel, in proving the unprovability of the Gödel sentence, has proven a new mathematical *truth*, this does not mean, as I have argued in the Politics of Logic, that what Wittgenstein says cannot be seen as itself suggesting a kind of realism with respect to mathematical *problems* (essentially, on the paradoxico-critical rather than the generic side of the distinction between the two post-Cantorian arguments). For instance, in a helpful reading of these remarks, Putnam and Floyd suggest that Wittgenstein can be seen as anticipating the thought that Gödel's result may show (only) that there is no model of ZFC that includes (only) the natural numbers. Though this suggests that Gödel's result does not after all establish any truths about a "realm" of the natural numbers themselves supposedly given in advance, it is nevertheless a telling meta-formal result *about models*, and one that must apparently have a "realist" construal if its real significance is to come into view. (cf. my "Badiou, Mathematics, and Model Theory").

Aristotle to Kant, that locates its basis in the activities, procedures and capacities of a thinking subject of consciousness.

IV

I have suggested that what I have called meta-formal realism provides a rigorous and appropriate basis for a development of Heidegger's own problematics of sense and time; besides providing for an underlying realism with respect to these structures and indeed to the question of givenness itself, it relates them to some of the most significant developments of contemporary formal reflection. The question may here arise, though, whether any such application of formal methodology (or methodology developed in accordance with the results and techniques of modern, symbolic logic) can really be made with respect to what Heidegger calls "fundamental ontology" or (later) "the history of Being" at all. For did not Heidegger himself resolutely and repeatedly oppose the application of the "empty" and "merely caulculative" methods of formal, symbolic logic or "logistics" to the question of Being itself? As I have noted, my attempt in this book is not primarily to develop an exegetically faithful reading of Heidegger, but rather to contribute to the development of several interrelated problems that he first pointed out, so it is a matter of relative indifference whether the specific kind of position that I have summarized as metaformal realism can indeed be attributed to Heidegger himself. Nevertheless, it is worth briefly considering the substance of his critique of the application of formal methods to ontology in order to more completely specify the underlying problematics themselves.

It is certainly true that Heidegger often, and throughout his career, opposes any conception according to which the techniques and methods of formal/symbolic logic, for instance of the kind developed by Frege, Russell and Whitehead, can by themselves determine ontological questions or clarify ontological problems. Already in the very early 1912 article "Recent research in logic," for example, Heidegger suggests that calculative "logistics" of the sort developed by Russell in *The Principles of Mathematics* is characterized by inherent "limits" in that it tends to "conceal the *meanings* of concepts and their *shifts in meaning*," thus leaving "the deeper sense of principles…in the dark". Logistics in this sense, according to Heidegger, is "simply not familiar with the problems of the theory of judgment" and its "mathematical treatment of logical problems" thus reaches "limits at which [its] concepts and methods fail, more precisely, there where the conditions of [its] possibility lie."[36] In Heidegger's subsequent work, the dominance of logistics (sometimes identified or associated with "positivism") and its substitution for "true" logic is often seen as, more broadly, representative of a broader regime of "calculative thinking" which is characteristic of the contemporary epoch of technology and its privileging of the real in the sense of "actuality" [wirklichkeit]. A passage from the Nietzsche lectures of 19—may be considered typical of this:

> The precedence of what is real [wirklich] furthers the oblivion of Being. Through this precedence, the essential relation to Being which is to be sought in properly conceived thinking is buried. In being claimed by beings, man takes on the role of the authoritative being.
>
> As the relation to beings, that knowledge is adequate which is used up by reification in accordance with the essential manner of beings, in the sense of the real as calculable and

[36] Nevertheless, in the article Heidegger praises Frege's work, especially in "On Sense and Reference" and "On Concept and Object" as "not yet appreciated in their true significance, let alone exhausted," and as essential not only for "any philosophy of mathematics" but also for "a universal theory of the concept." (p. 33)

> ensured. Knowledge thus becomes calculation. The sign of the degradation of thinking is the elevation of logistics to the rank of true logic. Logistics is the calculable organization of the unconditional lack of knowledge about the essence of thinking, provided that thinking, essentially thought, is that projecting knowledge which unfolds in virtue of Being in the preservation of truth's essence.

Heidegger thus connects the "elevation" of logistics in the sense of calculation to the status of a "true logic" with the more general "precedence" of the real which involves a conception or interpretation of all that is real in being in terms of its capacity to act on and affect beings. This regime is prepared, according to Heidegger, from long ago by the metaphysical interpretation of Being in terms of beings and by the privileging of "thatness", "reality," or "actuality" as the basic character of beings. Within this interpretation, Heidegger suggests, the techniques of mathematical "calculation" or "construction" attain the significance of demonstrating the existence of "something effective within a context of calculative proof." These techniques of calculation and construction thus become the basis for the constitution of the idea of effective causality that underlies "modern" physics and technology and thereby comes to dominate the knowledge and practices of the modern age. With this dominance of the actual in the sense of causally acting and effecting, the "essential determination" of the history of Being is "carried out to its prefigured completion."[37]

Heidegger thus sees the calculative techniques of symbolic and mathematical logic as, on the one hand, "empty" with respect to the actual structure and nature of presence and presencing themselves and, on the other, symptomatic in their growing dominance of the "metaphysical" conception of Being in terms of beings as it moves toward completion. The position is in a certain way overdetermined with respect to the actual "content" of the techniques of mathematical logic themselves: though these techniques are in themselves empty and incapable of supporting "thinking, essentially thought", nevertheless their

[37] The usual name for thatness, existence, testifies to the precedence of Being as *actualitas* in this interpretation. The dominance of its essence as *reality* determines the progression of the history of Being, throughout which the essential determination once begun is carried out to its prefigured completion. The real is the existing.
The existing includes everything which through some manner of causality *constituitur extra causas*. But because the whole of beings is the effected and effecting product of a first producer, an appropriate structure enters the whole of beings which determines itself as the co-responding of the actual produced being to the producer as the highest being. The reality of the grain of sand, of plants, animals, men, numbers, corresponds to the making of the first maker.
It is at the same time like and unlike his reality. The thing which as reality can be experienced and grasped with the senses is existent, but so is the object of mathematics which is nonsensuous and calculable.
"M exists" means: this quantity can be unequivocally constructed from an established point of departure of calculation with established methods of calculation. What is thus constructed is thus proven as something effective within a context of calculative proof. "M" is something with which one can calculate, and under certain conditions must calculate. Mathematical construction is a kind of constitution of the *constituere extra causas,* of causal effecting.

contemporary dominance, in connection with the regime of technology that they make possible, points in an important and even privileged way to what is most preeminently to be thought today. Despite this air of overdetermination, though, one might easily conclude from what Heidegger says that no methodology or result that essentially depends on formal or mathematical logic can play any positive role in furthering the ontological problematic itself, either in the sense of the "fundamental ontology" of Dasein or in the later sense of the history of being.

The methodology of meta-formal reflection that I have discussed, and which is modeled by Gödel's reasoning about the implications of his own results, does in fact depend essentially and in an obvious sense on the techniques of symbolic logic and mathematical proof; and so it might be thought, along these lines, that it just cannot be applied to the ontological problematics with which Heidegger is concerned. But in fact, none of the considerations that Heidegger introduces bear in any substantive way against the application of metaformal reasoning that I have suggested here.

First, as we have seen, what is in view with the kind of metaformal reasoning that I have discussed is not at all simply the mechanical application of a "formal" technique of symbol-manipulation, but rather a reflective illumination of the very conditions under which any such logical technique is possible and gains any possible relationship with truth. This reflective illumination, as we saw also in connection with the twofold consideration of truth and meaning in chapter 3, may more closely be compared to the task of what was traditionally called "transcendental" (rather than formal) logic in its evincing of the structure of the givenness of things themselves. But second, and more importantly, far from simply applying an effective technique of empty calculation that is assumed to have universal scope in itself, the "limitative" results of Gödel and Turing point exactly to the formally inherent limits of the actual effectiveness of any such technique. As such, they are themselves formally diagnostic of the configuration of thought and practice that simply *assumes* in advance the unlimited applicability of calculative techniques. Indeed, by demonstrating the necessary existence of the undecidable, the uncalculable, and the ineffective that accompanies *any* formal definition of technical or regular effectiveness, they also provide formally motivated terms for the fundamental critique of this configuration. This result of formally based reflection on formal methods – whereby these methods are inherently limited, in their relationship to truth, by an essential *ineffectivity* that necessarily accompanies them wherever they are applied – is anticipated in detail (as we have seen in chapter 1, above) by Frege's own conception, in opposition to the dominant psychologism, of logic as the site of an insistence of what is (precisely) real without being actual in the sense of "effective." But the inherent ineffectivity accompanying any total or calculative regime of thinking is only really rigorously demonstrated and positively verified, as we have seen, by the paradoxical and limitative results (including Russell's paradox, Gödel's theorems, and Turing's argument) that follow in quick succession from the completion of the "foundationalist" project itself.

In this respect, again, far from being opposed to Heidegger's consideration of the role of the dominance of "calculative" thought and its assumption of unrestricted applicability in the history of Being, the metaformal results of Gödel and Turing in fact confirm Heidegger's critique and point in a formally rigorous way to the very "closure" of the metaphysical regime of "actuality" that Heidegger himself attempts to describe. Here, it is thus not necessary to oppose the thinking that emerges from reflection on the scope and limits of formal/symbolic logic to the Heideggerian ontological problematic; rather, given the specific positive character of the limitative results that arise from this reflection, they can be seen as directly contributing to the development of this problematic and even confirming it by other means. Heidegger's own animadversions against the usefulness of symbolic logic (or the assumption of its unlimited applicability) are thus no reason to reject the application of metaformal reasoning I have suggested here. Aside from this, though, are there any positive arguments to be found in Heidegger's *corpus* that suffice to establish that formal reasoning of a "logical" or "mathematical" character *cannot* shed light on phenomenological or ontological issues?

By contrast with statements simply asserting the "emptiness" of formal/symbolic logic, or genealogical/historical descriptions of what Heidegger sees as the role of "logic" as such (and primarily in its Aristotelian or Hegelian forms) in the development and fixation of the metaphysical tradition, such positive arguments are much harder to find in Heidegger's texts. One such, however, is suggested in the course of a critical discussion of Husserl's phenomenology in Heidegger's (early) Freiburg lecture course, "Ontology: The Hermeneutics of Facticity," from the summer of 1923. Here, Heidegger challenges what he sees as Husserl's presupposition of "mathematics and the mathematical natural sciences" as a *model* "for all scientific disciplines," which according to Heidegger suggested, in the earlier development of Husserl's phenomenology, that phenomenological description itself be "[elevated]…to the level of mathematical rigor:"[38]

> Nothing more needs to be said here about this absolutizing of mathematical rigor. This is not the first time it has surfaced, but rather it has for a long time dominated science, finding an apparent justification in the general idea of science which appeared among the Greeks, where one believed that knowledge was to be found in knowledge of universals and - what is seen to be the same thing - knowledge of what is universally valid. But this is all a mistake. And when one cannot attain such mathematical rigor, one gives up.
>
> Fundamentally, one does not even realize that a prejudice is at work here. Is it justified to hold up mathematics as a model for all scientific disciplines? Or are the basic relations between mathematics and the other disciplines not thereby stood on their heads? Mathematics is the least rigorous of disciplines, because it is the one easiest to gain access to. The human sciences presuppose much more scientific existence than could ever be achieved by a mathematician. One should approach a scientific discipline not as a system of propositions and grounds for justifying them, but rather as something in which factical Dasein critically confronts itself and explicates itself. To bring mathematics into play as the model for all scientific disciplines is

[38] This attribution of this position to Husserl is in fact puzzling in at least two ways. First, of course, given Husserl's longstanding and decisive critique of naturalism and the natural attitude, it can hardly be said (whatever the role of mathematics itself in serving as a model for phenomenological description) that he *generally* privileged "mathematical natural science" as a model for phenomenological investigation. But second, although it is indeed suggested in the *Logical Investigations* that mathematics in the sense of a "mathesis universalis" can serve as a formal structure for all *logical* theory, by 1923 Husserl had already clearly rejected the idea that the *phenomenological* structure of experience itself could always be mathematized in a formally exact way: see, e.g. Ideas I (1913) section ---.

> unphenomenological- the meaning of scientific rigor needs rather to be drawn from the kind of object being investigated and the mode of access appropriate to it.

According to this argument, in other words, it is inappropriate to treat mathematics as the "model" for the phenomenological description of what is given in experience, or methodologically to impose the kind of rigor that is characteristic of it here. This is because, as Heidegger argues, phenomenology is not a topical area or a categorical field but rather a method of developing the "how" of access into what is present in intuition, just as it gives itself to experience there. Since it is concerned with the mode of access in this way, phenomenological description has to be developed according to the kind of access that is characteristic of the particular field or kind of object being investigated in each case, and it is accordingly a mistake to take the characteristic universality and universal transmissibility of mathematical knowledge as a methodological or thematic model for all "scientific" inquiry. In this respect, in fact, Heidegger suggests, this characteristic universality and accessibility of mathematics makes it in fact the "least rigorous" of disciplines, in that it means that it fails to involve the complexity or singularity of the "scientific existence" that the human sciences themselves presuppose and attempt to theorize.

From the perspective of meta-formal reflection that I have suggested here, it should be said, first, that there is no need to presuppose the purported "universality" and accessibility of mathematical *objects* in order to apply the lessons of "metalogical" or "metamathematical" reflection to the problems of (phenomenological) access and givenness. As we have seen, the attitude of meta-formal realism should, on the one hand, be sharply distinguished from the (vulgar) "Platonist" attitude of assuming or presupposing the timeless existence of a range of mathematical objects universally accessible due to their privileged residence in a kind of *topos ouranous* quite alien to anything specifically involved in "our" form of life; while, on the other, the positive results on which meta-formal realism turns provide grounds for a formally based reconsideration of what is involved – in the theory of proof, the force of rules of inference, and the provision of axioms themselves – in anything that can reasonably be seen as the "accessibility" of mathematical "objects" to begin with.

Second, though, and along the same lines, though, it should also be asked what kinds of accessibility *do* characterize mathematical knowledge, and what is the form underlying these kinds of accessibility in the facticity of a life, here determined not simply in terms of any factual-anthropological conception of the "human" but in a way structurally corresponding to its proper modes of givenness and presence themselves. For mathematics is after all, among other things, an activity undertaken in the course of such a life among other activities of theoretical reflection and practice; and without yet assuming anything determinate about the ontological mode of existence of its objects, it is certain that the problem of access here raises quite specific and difficult problems which must be confronted by *any* phenomenological or ontological theory of givenness or presence as such. Especially in connection with the idea of the infinite, which receives (as we have seen) a fundamental and transformative articulation in the work of Cantor and the developments which follow him, these are problems of "access" that are not in fact limited to the "philosophy of mathematics" in a narrow sense, but rather raise questions bearing on the structural form of "our" mode of life (for instance, the nature and meaning of its long-discussed "finitude") itself. As I have tried to argue here, there are also not distinct from the problems constitutively involved in any account of “our” access to *meaning* or *sense* and indeed of its own basic constitution, insofar as this basic constitution *always* involves the “infinite” character of the one over the (unlimited) many. These are the problems visibly taken up in an original fashion (although not resolved) by Plato in the heroic dialectics of his late attempts at a revision of the classical “theory of forms;” and, as I have tried to show (especially chapter 2) they are also problems that can by no means be avoided by an ontological hermeneutics in its own development of the question of access and accessibility, most of all where this question overlaps with the problem of truth. Here, indeed, as I shall attempt to demonstrate over the next several chapters, the insistence of these problems points in a basic structural way to the original problem of the givenness of time, insofar as it can be experienced or measured at all.

In particular: hermeneutic attention to the formal basis and ontological constitution of constituted time will verify that time as such is never just “there” before us to be counted by means of an activity or process of an agent simply external to it. Rather, as I shall attempt to show, it is always *doubled* as both counting and counted, as the experienced and the thinkable, as hence as the time of the world in which we live and the time of sense with which we think. Metaphysical thought opposes the “realm” of the thinkable and that of the experienceable as the timeless and the intra-temporal; but the ontological/hermeneutic formal interpretation of the basis of the possibility of countable time will reveal the more original problematic horizon of this determination. Since Aristotle, and up to Kant, metaphysics also thinks the basis of counting, hence the basis of "experienced" time, in the self-relational activity of a self or subject which also constitutes its original temporal form. But there is also a genesis of sense, and of the being of number, that owes nothing to the subject. This will becomes clear in relation to what Plato thought as the ideal genesis of number, where it points to an original problematic of the finite and the infinite in relation to the ideal conditions of being and becoming that gains resonance once more through the development of the metalogical problematic today.

For these reasons, over the next several chapters of the investigation, we shall take up the old problem of “mathematical existence” on a renewed ontological-hermeneutic ground, not with a view to establishing or securing the model of mathematical objectivity as absolute existence, but as the concrete problem of the form of sense as it communicates with the structure of temporality and with presence “in general.” The investigation will lead us to consider such matters as the possible “givenness” of the infinite to thought, the peculiar temporal character of “historical” languages which are nevertheless capable of expressing judgments and truths “once and for all,” and the mysterious thought of a superior “ideal genesis” of forms that Plato appears to assay, in his last writings and unwritten doctrines, on the model of an actual origin of numbers from the superior principles of the one and the unlimited many themselves. The aim will be a substantial clarification, relevant to the contemporary “ontological” situation, of the problem of the sense and truth of Being insofar as it comes to light as time.